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Preferential information dynamics model for online social networks

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HIGHLIGHTS

- A spreading model for dissemination of preferential information is proposed.
- Double infection rate parameters are adopted in the proposed model.
- The proposed model achieves good results in BA networks of various sizes.

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ABSTRACT

In recent years, online social networks have become an important site for companies to promote their latest products. Consequently, evaluating how many clients are affected by preferential information distributed in online social networks has become essential. In this paper, a novel dynamic model called the follower super forwarder client (FSFC) model is proposed to address the spreading behavior of preferential information in online social networks. The mean field theory is adopted to describe the formulas of the FSFC model and the key parameters of the model are derived from the past forwarding data of the preferential information. The edge between a large-degree node to a small-degree node has a greater weight. In addition, two kinds of infection probabilities are adopted for large-degree node forwarders and small-degree node forwarders. To evaluate the performance of the FSFC model, preferential data published on the Sina microblog (www.weibo.com) for the Vivo smartphone, Alibaba's Tmall shopping site, and the Xiaomi phone were selected as real cases. Simulation results indicate that the relative errors of the output of the FSFC model compared with the actual data are 0.0068% (Vivo smartphone), 0.0085% (Tmall), and 0.032% (Xiaomi phone), respectively. The results verify that the FSFC model is a feasible model for describing the spreading behavior of preferential information in online social networks.

1. Introduction

Humans live in a world replete with complex networks [1]. The spreading dynamics of these complex networks are attracting increasing interest from researchers as regards their communication mechanism, dynamic behavior in society and nature, and the feasibility of deriving control methods for them [2–4].

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Table 1
Summary of related work involving improvement of the classical SIS and SIR models.

Classification	Model	Highlights
Dissemination rules modification	SIR [11]	At every time step, any ignorant node can become an independent spreader with a certain probability.
	SIRS [15]	Recovered nodes are considered in a proposed SIRS-based epidemic model with a feedback mechanism on heterogeneous networks.
	SIS [16]	The effects of the linear combination of multiple propagation media on propagation are considered.
	SIQRS [17]	A new quarantine individual (Q) is incorporated into the SIR model.
	New SIQRS [18]	Nonlinear infectivity is incorporated in a new SIQRS model.
	SIS [19]	The communicator simply passes some of the information to its neighbor. The recipient is required to obtain complete information from different communicators.
	SIS [20]	The susceptible are spontaneously infected with a certain probability at any time during the spread.
	SAIRS [21]	An empty state is proposed: the node in the empty state changes to state S with a certain probability, and the states S, A, I, and R become empty with a certain probability.
Infection rates modification	SIS [13]	An SIS model with delay denoting the average incubation period of the disease in a vector is proposed.
	SIS [22]	A nonlinear incidence rate is used to respond to psychological or inhibition effects.
	SIS and SIR [23]	A dynamical model that uses the decrease in infection rate caused by immunity and heightened vigilance is proposed.
	SIRS [24]	Infection in the network is time-delayed.
	SIB [12]	The susceptible become a communicator with a different probability.

Two classical models, namely, susceptible infected susceptible (SIS) and susceptible infected recovered (SIR), have been proposed for the spreading dynamics (or network propagation dynamics) based on the mean field theory [5–8]. Both models were originally used to explain the spreading of diseases [9,10]. Recently, a new model based on SIS and SIR has been developed to describe the spreading dynamics of complex networks (Table 1). The classical SIS and SIR models have been improved either by modifying the rules of dissemination or the infection rates. For instance, Li et al. [11] proposed a SIR rumor model that incorporates independent spreaders. In their new model, at every time step, any ignorant node can become an independent spreader with a certain probability. Li et al. [12] proposed a susceptible infected beneficial model based on scale-free networks. They assume that susceptible individuals are infected by an infected individual or a beneficial individual with various probabilities and incorporate immigration and emigration into the spreading rules. Kang et al. [13] presented an SIS model with delay for scale-free networks in which the function of infected vectors is considered. Chu et al. [14] considered the effect of weighted network and nonlinear infection rate on epidemic infection and stated that the infection rate is related to the degree of the node.

It has been discovered that most real networks are actually scale-free networks; further, the scale-free feature is an important aspect of social networks [25,26]. Newly added nodes in a network always connect to the nodes already existing in networks that have a large degree [25]. For example, business groups and news media belong to large-degree nodes ranging in size from millions to tens of millions, and they have a huge influence on ordinary users in social networks [27,28]. In contrast, ordinary users belong to small-degree nodes, and they have miniscule influence on the other ordinary users in social networks. Hence, with rapid development of social networks and new media, increasingly more business groups are choosing to promote their products by publishing preferential information through social media. Hence, studying the spreading dynamics of preferential information for online social networks is an important issue. However, traditional dynamic models such as SIR, SIS, and their improved versions cannot be adopted to explain the dynamic behavior of preferential information spread. The spreading of preferential information is not infinite; it has a very limited range. The survival period of preferential information published by social media is typically several days (or weeks) and the preferential information is forwarded two or three times in most cases [29].

In this paper, we propose a new dynamic model based on the SIR model, called the follower super forwarder client (FSFC) model. In the FSFC model, we introduce new rules for the spreading process of preferential information on social networks. We make two assumptions for the proposed model: 1. Small-degree nodes are more likely to forward information from large-degree nodes. 2. The edge between a large-degree node to a small-degree node has a greater weight. In a general way, celebrities can always influence the behavior and opinions of many ordinary people in social networks. However, ordinary people are rarely affected by other ordinary people. The FSFC model can be set using different values for parameters β_1 and β_2 to adapt to different propagations. The preferential data of the Vivo smartphones, Alibaba's Tmall shopping site, and the Xiaomi phones are used as real cases to estimate the performance of the proposed FSFC model. Simulation results show that

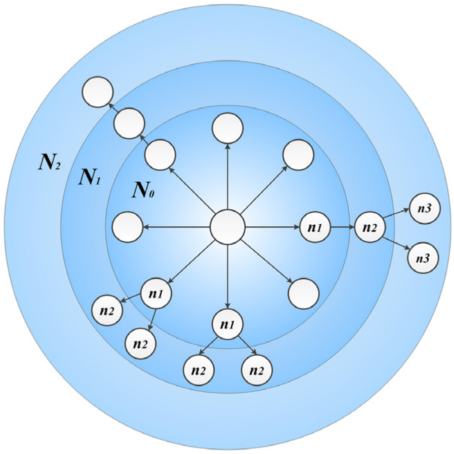

Fig. 1. Preferential information spread on an online social network. The center node represents the super-communicators where the linked nodes receive the information.

the output of the FSFC model is congruent with the actual data of the three cases. Hence, the FSFC model can be a feasible dynamic model for preferential information.

2. Empirical observation

In order to discover the patterns of information diffusion consider the network structure presented in Fig. 1. The central node (Fig. 1) is often represents the source of information. In the spreading of preferential information, the source of information is always the center of the graph node, and it represents a super-communicator with numerous followers. In microblog networks, super-communicators are predominantly composed of celebrities, commercial accounts, and news media. This paper presents the preferential information spreading model; i.e., preferential information released by several commercial accounts. The main purpose of preferential information release by commercial accounts is to promote their products. Thus, such preferential information is often accompanied by a lottery, with forwarding of the information enabling participation in the lottery to win prizes. Consequently, the information will be forwarded multiple times.

However, it is noteworthy that mass forwarding of preferential information mainly consists of a large number of first-level forwarding, and only a few second-level or multi-level forwarding. The first-level forwarding means that the followers are directly forwarded from the source blog of the commercial account, as shown in the N_0 layer in Fig. 1. The second-level forwarding means that the user is forwarded information from their friends, as shown in the N_1 layer. Third-level forwarding occurs at level N_2. Sometimes the publishers of preferential information microblogs will forward their microblog themselves, in order to cause more users to forward it. There are a few cases in which other large-degree nodes reforward this preferential information to expand the scope of information spread. However, we disregard such a scenario in this model because the propagation starting from a new large-degree node has a similar spreading situation as before, and also has a large number of first-level transports (forwarding the microblog as the original microblog released by the new large-degree node) and a small number of multi-level transponders . This is depicted in Fig. 2. Therefore, we can simplify a variety of situations into the simplified model shown in Fig. 1.

3. Standard SIR model

In order to describe the spreading dynamics of preferential information, we propose the FSFC model based on the SIR model [6,30]. In the SIR model, individuals can be divided into three states: susceptible, infected, and removed. Considering the heterogeneous structure of the network, we use $S_k(t)$, $I_k(t)$, and $R_k(t)$ to represent the density of the three states, with degree k at time $t.P\left(k'|k\right) I_{k'}\left(t\right)$represents the average of the joint probability of a node connected from a degree k to a node of arbitrary degree k'. The density of the three states conform to the normalization equation: $S_k\left(t\right) + I_k\left(t\right) + R_k\left(t\right) = 1$. Fig. 3 demonstrates the relationship among these three categories.

In the SIR model, the disease begins to spread from multiple infected individuals. At first, most individuals are susceptible individuals. After contact with susceptible individuals, individuals who are infected transmit the disease to susceptible individuals at probability λ. Infected individuals may either be cured or die with probability μ to become removed. The

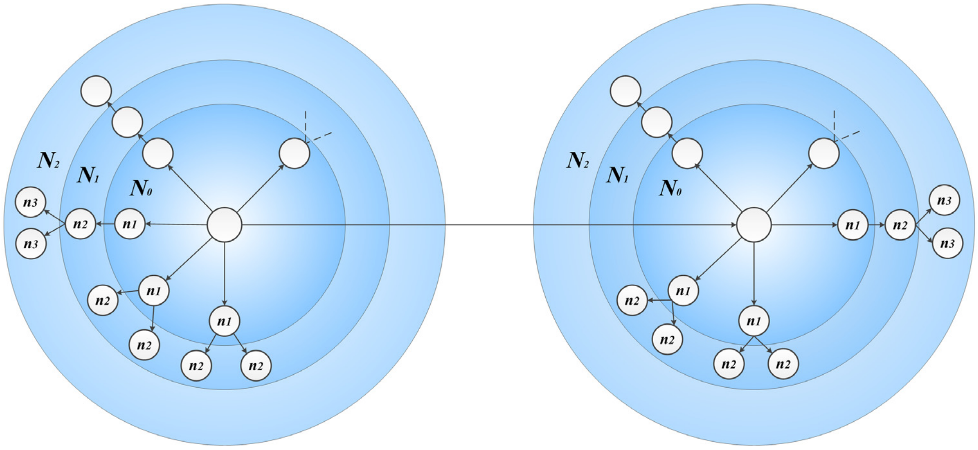

Fig. 2. Structure of the preferential information dissemination forwarded by large-degree nodes. Two large-degree nodes generate very similarly propagation structures.

Fig. 3. Flow diagram illustrating the SIR model.

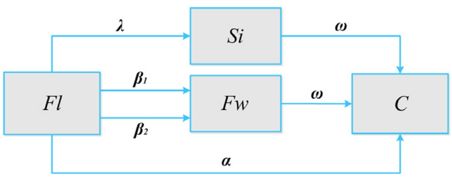

Fig. 4. Flow diagram of the FSFC model.

dynamic equation according to the SIR model is as follows:

$$\frac{dS_k(t)}{dt} = -\lambda k S_k(t) \sum_{k'} P\left(k'|k\right) I_{k'}(t) \tag{1}$$

$$\frac{dI_k(t)}{dt} = \lambda k S_k(t) \sum_{k'} P\left(k'|k\right) I_{k'}(t) - I_k(t) \tag{2}$$

$$\frac{dR_k(t)}{dt} = I_k(t) \tag{3}$$

The above three equations represent the changes in susceptible, infected, and removed state density, respectively.

4. FSFC model

In the FSFC model, individuals are divided into four categories: followers, super, forwarders, and clients. Followers are individuals who never know the preferential information. A super is an individual who releases the preferential information. Forwarders are individuals who know and forward the preferential information. Clients are individuals who know the preferential information but do not forward it. Fig. 4 illustrates the spreading rules of the FSFC model. The preferential information is initially issued by a large-degree node, which is the first forwarder node. Initially, most individuals are followers. During the preferential information spreading process, the small-degree nodes are more likely to forward the preferential information issued by the large-degree nodes than the preferential information from nodes whose degree is equal to theirs. Furthermore, a follower is infected by a super with an infection probability β_1, whereas a follower is infected by a forwarder with an infection probability β_2. In addition, the follower is converted to client with probability α. Further, a forwarder and super are converted to client with probability ω, which is usually equal to one.

Considering the heterogeneous structure of the network, we use $\mathrm{Fl}_k(t)$, $\mathrm{Si}_k(t)$, $\mathrm{Fw}_k(t)$, and $\mathrm{R}_k(t)$ to represent the density of the four states with degree k at time t. For the different degrees, the density of the four states conform to the normalization equation $\mathrm{Fl}_k(t) + Si_k(t) + \mathrm{Fw}_k(t) + \mathrm{C}_k(t) = 1$.

The spreading rules of the FSFC model are as follows:

$$\begin{aligned}\frac{dFl_k(t)}{dt} = &-kFl_k(t)\sum_{k'}P\left(k'|k\right)Si_{k'}(t)\left(k'-k\right)\beta_1 - kFl_k(t)\sum_{k'}P\left(k'|k\right)Fw_{k'}(t)\left(k'-k\right)\beta_2\\ &-kFl_k(t)\sum_{k'}P\left(k'|k\right)Si_{k'}(t)\lambda - kFl_k(t)\sum_{k'}P\left(k'|k\right)Si_{k'}(t)\alpha - kFl_k(t)\sum_{k'}P\left(k'|k\right)Fw_{k'}(t)\alpha\end{aligned} \tag{4}$$

$$\frac{dFw_k(t)}{dt} = kFl_k(t)\sum_{k'}P\left(k'|k\right)Si_{k'}(t)\left(k'-k\right)\beta_1 + kFl_k(t)\sum_{k'}P\left(k'|k\right)Fw_{k'}(t)\left(k'-k\right)\beta_2 - \omega Fw_k(t) \tag{5}$$

$$\frac{dSi_k(t)}{dt} = kFl_k(t)\sum_{k'}P\left(k'|k\right)Si_{k'}(t)\lambda - \omega Si_k(t) \tag{6}$$

$$\frac{dC_k(t)}{dt} = kFl_k(t)\sum_{k'}P\left(k'|k\right)Si_{k'}(t)\alpha + kFl_k(t)\sum_{k'}P\left(k'|k\right)Fw_{k'}(t)\alpha + \omega \mathrm{Fw}_k(t) + \omega Si_k(t) \tag{7}$$

where k is the degree of the node. $P\left(k'|k\right)$ represents the average joint probability of a node connected from a degree k to a node of arbitrary degree k'. $P\left(k'|k\right) = k'P(k')/\langle k\rangle$ [31].

5. Steady-state analysis

Preferential information is started by a node, and forwarded through the fans to get the spread. In this model, we assume that there is only one super at the beginning, and the rest are followers. Thus, the initial condition for preferential information spreading is $\mathrm{Si}(0) \approx 0$, $\mathrm{Fl}(0) \approx 1$, and $\mathrm{Fw}(0) = \mathrm{C}(0) = 0$. Subsequently, supers will remain silent while the number of transferees will increase. Then, the number of forwarders will decrease to zero. During this period, the number of customers will continue to increase. In the final equilibrium state, only followers and clients remain.

$$\begin{aligned}\frac{dFl_k(t)}{dt} = &-kFl_k(t)\sum_{k'}P\left(k'|k\right)Si_{k'}(t)\left(k'-k\right)\beta_1 - kFl_k(t)\sum_{k'}P\left(k'|k\right)Fw_{k'}(t)\left(k'-k\right)\beta_2\\ &-kFl_k(t)\sum_{k'}P\left(k'|k\right)Si_{k'}(t)\lambda - kFl_k(t)\sum_{k'}P\left(k'|k\right)Si_{k'}(t)\alpha - kFl_k(t)\sum_{k'}P\left(k'|k\right)Fw_{k'}(t)\alpha\\ \frac{dFw_k(t)}{dt} = &\ kFl_k(t)\sum_{k'}P\left(k'|k\right)Si_{k'}(t)\left(k'-k\right)\beta_1 + kFl_k(t)\sum_{k'}P\left(k'|k\right)Fw_{k'}(t)\left(k'-k\right)\beta_2 - \omega Fw_k(t)\\ \frac{dSi_k(t)}{dt} = &\ kFl_k(t)\sum_{k'}P\left(k'|k\right)Si_{k'}(t)\lambda - \omega Si_k(t)\end{aligned}$$

Consider the diffusion of a tweet on online social networks. The entire population involved with the super spreader comprises most of the users, and very few users unaware of the event. Therefore, Fl can be zero. Here, we assume that the equilibrium state after an event with the super spreader is (Fl, Fw, Si, C) $\approx$(0, 0, 0, 1).

Note that $\mathrm{C_K}(t)$ in the differential equation, Eq. (7), is uncoupled from the other three equations. With the equilibrium and the first three equations above, we can compute the Jacobian matrix with equilibrium (Fl, Fw, Si, C) $\approx$(0, 0, 0, 1):

$$\mathrm{J}(\mathrm{Fl}, \mathrm{Fw}, \mathrm{Si}) = \begin{bmatrix} \mathrm{A}_1 & \mathrm{B}_{12} & \mathrm{B}_{13} & \cdots & \mathrm{B}_{1\mathrm{K}_{\max}} \\ \mathrm{B}_{21} & \mathrm{A}_2 & \mathrm{B}_{23} & \cdots & \mathrm{B}_{2\mathrm{K}_{\max}} \\ \vdots & \vdots & \vdots & & \vdots \\ \mathrm{B}_{\mathrm{K}_{\max}1} & \mathrm{B}_{\mathrm{K}_{\max}2} & \mathrm{B}_{\mathrm{K}_{\max}3} & \cdots & \mathrm{A}_{\mathrm{K}_{\max}} \end{bmatrix}$$

When at equilibrium (Fl, Fw, Si, C) $\approx$(0, 0, 0, 1),

$$\mathrm{A_i} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\mathrm{w} & 0 \\ 0 & 0 & -\mathrm{w} \end{pmatrix}, \mathrm{B_{ij}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\mathrm{w} & 0 \\ 0 & 0 & -\mathrm{w} \end{pmatrix}$$

the characteristic equation of the Jacobian matrix is

$$(-1)^{3k-2}\lambda_0^{3k-2}(-\mathrm{w}-\lambda_0)^2 = 0,$$

The eigenvalues are $\lambda_1 = \lambda_2 = \lambda_3 = \cdots = \lambda_{k-2} = 0$ and $\lambda_{k-1} = \lambda_k = -w$. As there is no positive real part of all the features, (Fl, Fw, Si, C) $\approx$(0, 0, 0, 1) is locally stable.

Table 2
Vivo smartphone forwarding data.

	Primary forwarders	Secondary forwarders	Tertiary forwarders
Number	21 964	105	0

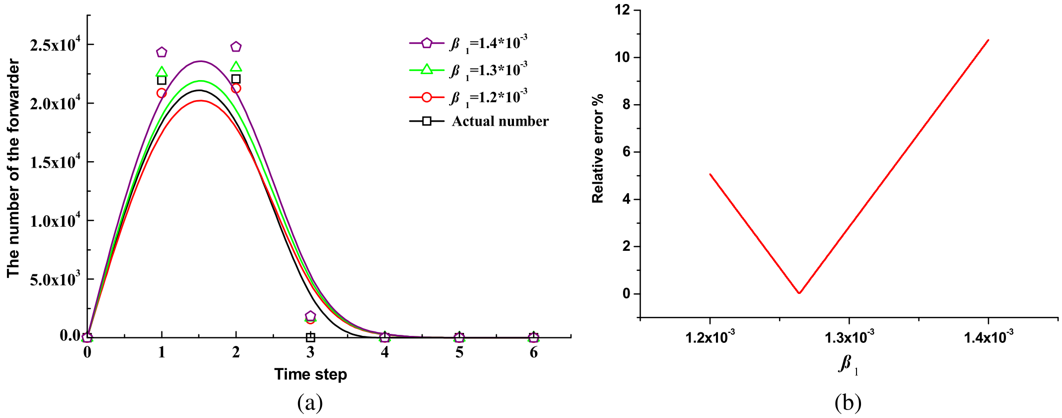

Fig. 5. Selection of β_1 : (a) Change in the number of forwarders as β_1 increases when $\beta_2 = 6.8 \cdot 10^{-4}$. (b) Change in relative error between the simulated and real values in the first time step as β_1 increases.

6. Selection of parameters for the FSFC model

6.1. Selection of $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$

Usually, for the same preferential information, there are primary forwarders, secondary forwarders, and tertiary forwarders. Primary forwarders directly forward the preferential information issued by commercial account. Secondary forwarders forward the preferential information issued by the primary forwarders. Tertiary forwarders forward the preferential information issued by the secondary forwarders. Table 2 shows the statistical data of different forwarders for the preferential information of the Vivo smartphone [32]. The data were collected from the Sina microblog (www.weibo.com).

Active users are individuals who use online social media frequently. According to the 39th China Internet Development Statistics Report, released by the China Internet Network Information Center [33], Sina microblog had 271.43 million users in 2016, and the proportion of active users was 37.1%. The number of followers for the Vivo smartphone was 17,727,899 [34]. We use the curve simulation method to obtain parameters β_1 and β_2. First, we insert various values of β_1 into Eqs. (4)–(7), and calculate the relative error between the actual value and the calculated value to obtain the optimal parameters. Eqs. (4)–(7) yield the ratio of each state, which must then be multiplied by the number of active users in the entire network to obtain the calculated value. Fig. 5 shows the β_1 selection process. The best value of β_1 obtained is $\beta_1 = 1.264 \cdot 10^{-3}$. By choosing different values, optimal solution of β_1 will eventually be obtained. We selected 201 points from 0.0012 to 0.0014 with 0.000001 increments and inserted them into the formula for calculation.

In the second step, the optimal value $\beta_1 = 1.264 \cdot 10^{-3}$ is inserted into Eqs. (4)–(7). Then, the value of β_2 is changed to calculate the relative error between the actual value and the calculated value as shown in Fig. 6, and the best value of β_2 is obtained $\beta_2 = 6.704 \cdot 10^{-4}$.

6.2. Selection of α, ω, and λ

The average degree of the scale-free network represents the number of followers that users of an online social network have. We assume that α is a reciprocal of average degree, which means that one of the fans of each forwarder will become a client. Further, we assume that the average number of users in the network is 30. Consequently, $\alpha = 1/30$. In our model, preferential information is only forwarded once by each user, and that user changes from forwarder to client after forwarding. Hence, we set $\omega = 1$. This is also consistent with reality because very few people repeatedly forward the same message.

Because there is only one super in our model, we need to set λ to a very small value $\lambda = 1 \times 10^{-6}$ to avoid the forwarder becoming a super. Otherwise, if λ is set to other values, it may be more general, such as multiple supers in the network.

6.3. Discussion

It is well-known that high infection rates inevitably lead to wider ranging infection. In this paper, β_1 and β_2 affect the spreading of preferential information, but their active function is not the same. Iribarren and Moro also proposed a double-prevalence model, in which they considered the impact of infection rates in the first and second steps [35]. As shown in

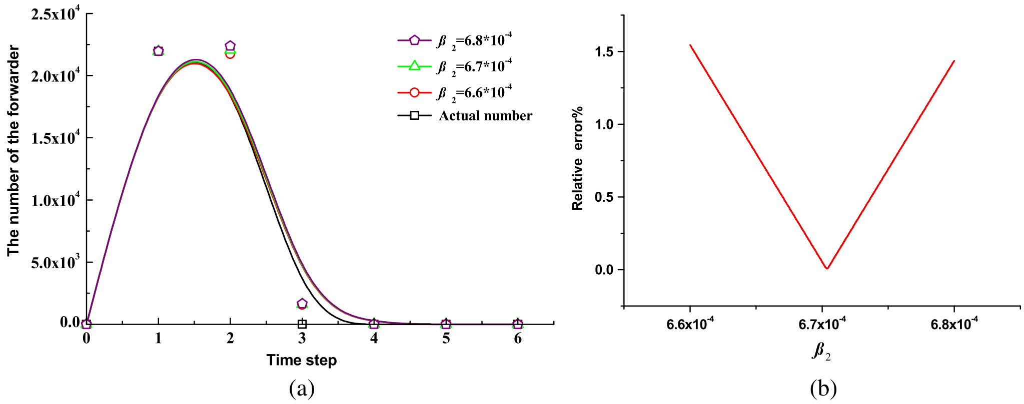

Fig. 6. Selection of β_2: (a) The number of forwarders changes with increasing β_2. when $\beta_1 = 1.264 \cdot 10^{-3}$ (b) Change in relative error between the simulated and real values in the second time step as β_2 increases. From (b), we select 201 points from 0.00066 to 0.00068 for calculation.

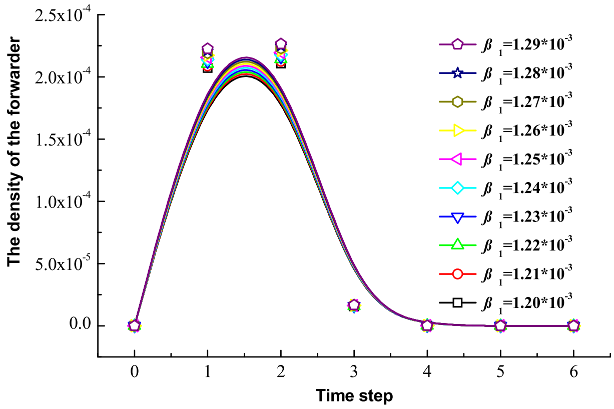

Fig. 7. Changes in the density of the forwarders when $\beta_2 = 6.8 \times 10^{-4}$. As the main route of propagation of information in the model is from large nodes to small nodes, the increase of β_1 will increase the number of forwarders in the entire network.

Fig. 7, β_1 takes effect in the early stages of the preferential information spreading, and β_2 takes effect in the latter part of the preferential information spreading as shown in Fig. 8. The reason is that the early stages of preferential information spreading communication are between the large-degree nodes and the small-degree nodes, whereas in the latter stages of preferential information spreading, communication is between the small-degree nodes. Further, the infected scope of preferential information is enlarged with increasing β_1 and β_2, as shown in Fig. 9.

7. Numerical simulation

7.1. Simulation environment and parameter setting

To evaluate the performance of the FSFC model, we collected data from online social networks (http://weibo.com/) and developed programs using MATLAB 2015b. We modeled a BA scale-free network and $\mathrm{P}(\mathrm{k}) = 2\,\mathrm{m}^2\mathrm{k}^{-3}$. Further, we assumed that only the commercial account published the preferential information initially; that is, only one super node was included in the online social network in the beginning. Hence, for the initialization phase, all the other states were approximately zero. Specifically, the super density is a reciprocal of all nodes and $\mathrm{Fl}_k(t) + Si(t) + \mathrm{Fw}_k(t) + \mathrm{C}_k(t) = 1$. For the FSFC model, the MATLAB programs were written according to Eqs. (4)–(7). The average density of supers, followers, forwarders, and clients was adopted as the performance indices of the two models, as shown in Eqs. (8)–(11).

$$Fl(t) = \frac{\sum Fl_k(t) \times N_k}{N} \tag{8}$$

$$Si(t) = \frac{\sum Si_k(t) \times N_k}{N} \tag{9}$$

$$Fw(t) = \frac{\sum Fw_k(t) \times N_k}{N} \tag{10}$$

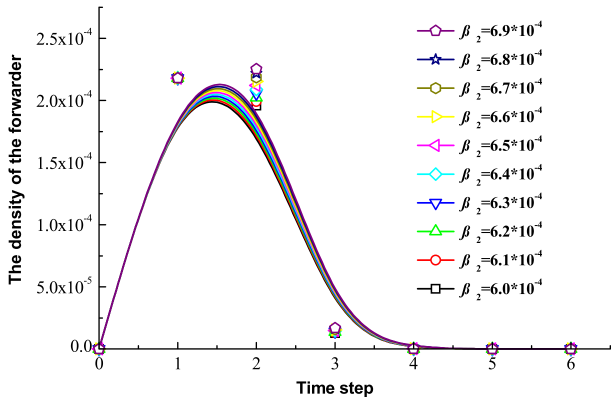

Fig. 8. Changes in the density of the forwarders when $\beta_1 = 1.264 \cdot 10^{-3}$. β_2 represents the probability of infection between nodes of the same degree level, which mainly affects the later spreading of information.

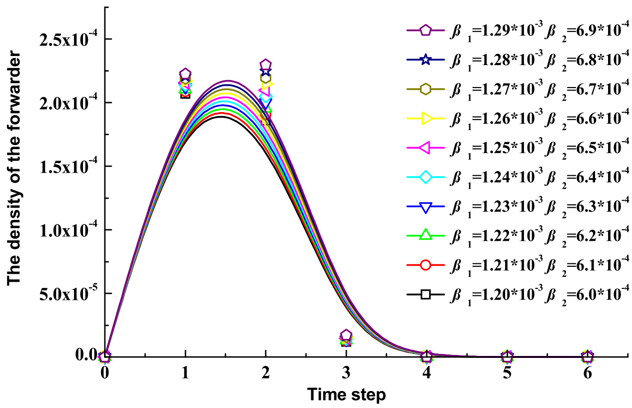

Fig. 9. Changes in the density of the forwarders for increasing β_1 and β_2. Increasing the β_1 and β_2 values at the same time will significantly increase the scope of the information.

$$C(t) = \frac{\sum C_k(t) \times N_k}{N} \tag{11}$$

where N_k represents the number of nodes for degree k and N represents the total number of nodes in the scale-free network.

7.2. Simulation results for Vivo smartphone preferential information spreading

As mentioned in Section 6, taking the Vivo smartphone as a case study, the parameters of the FSFC model were $\beta_1 = 1.264 \cdot 10^{-3}$ and $\beta_2 = 6.704 \cdot 10^{-4}$, and $\alpha = 0.033$. In the simulation for this example, a BA network with 51 811 points was used. The largest node degree was 1103 and the smallest was 10. Fig. 10 shows the simulation results for the relationship between the density of forwarders and clients with the forwarding time steps. It can be seen that the density of the forwarders goes through a process from rapid growth to rapid decline, and the whole spreading process ends in the fourth forwarding time step. Conversely, the density of clients increases rapidly in the first and second forwarding time steps and tends to stabilize after the third forwarding time step. The simulation results show that people like to forward the preferential information one or two times at most when they get the information from the commercial account and the density of the people who are interested in the preferential information does not increase because the preferential information has been forwarded more than three times.

Fig. 11 shows the variation of the number of clients calculated by the FSFC model and the corresponding actual number of clients. The number of users shown in the figure is calculated according to the following formula, where 100 700 530 is the result of multiplying the number of users on Sina Weibo by the proportion of active users (37.1%).

$$\textit{Actual number} = 100\,700\,530 \times \text{ density of Fw (k) or C(k)} \tag{12}$$

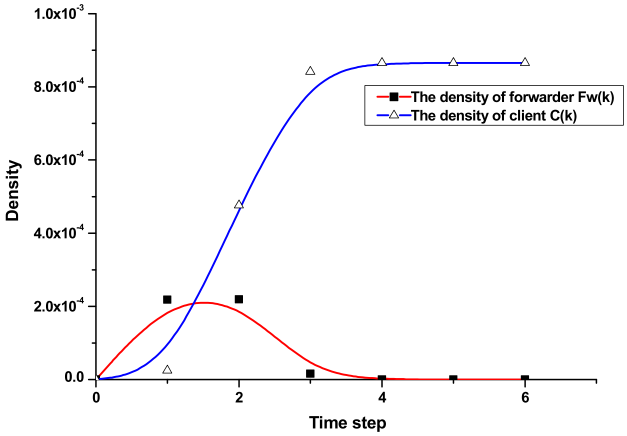

Fig. 10. Density variation of $Fw(t)$ and $C(t)$ with time step for the Vivo smartphone case. The red line shows the trend of forwarder density when a preferential information is released. The message is enthusiastically forwarded in the beginning and becomes unattractive eventually. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

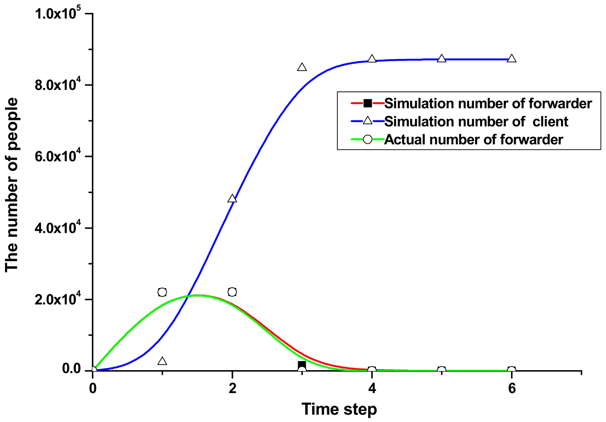

Fig. 11. Variation in the number of forwarders and clients with time step for the Vivo smartphone case. Simulation number of forwarder fits actual number of forwarder very well.

Table 3
Tmall forwarder data.

	Primary forwarders	Secondary forwarders	Tertiary forwarders
Number	33 226	12 116	1609

7.3. Simulation results for Tmall preferential information spreading

Table 3 lists the number of forwarders for the preferential information spreading of Alibaba's Tmall shopping site in social networks [36].

By fitting, the parameters of the FSFC model for Tmall are $\beta_1 = 1.293 \times 10^{-3}$, $\beta_2 = 1.358 \times 10^{-3}$, and $\alpha = 0.033$. In the simulation for this example, a BA network with 20,000 points was used. The largest node degree was 732 and the smallest was 10. Fig. 12 shows the simulation results for the relationship between the density of the forwarders and clients with forwarding time steps. Fig. 13 shows the simulation number of clients calculated by the FSFC model and the corresponding actual number of clients.

7.4. Simulation results for Xiaomi phone preferential information spreading

Xiaomi phone is one of China's most popular mobile phone brands. Table 4 lists the number of each type of forwarders for Xiaomi phone preferential information spreading in social networks [37].

By fitting, the parameters of the FSFC model for Xiaomi phone were $\beta_1 = 1.364 \times 10^{-3}$, $\beta_2 = 1.506 \times 10^{-3}$, and $\alpha = 0.033$. In the simulation of this example, a BA network with 10 000 points is used. The largest node degree is 505 and

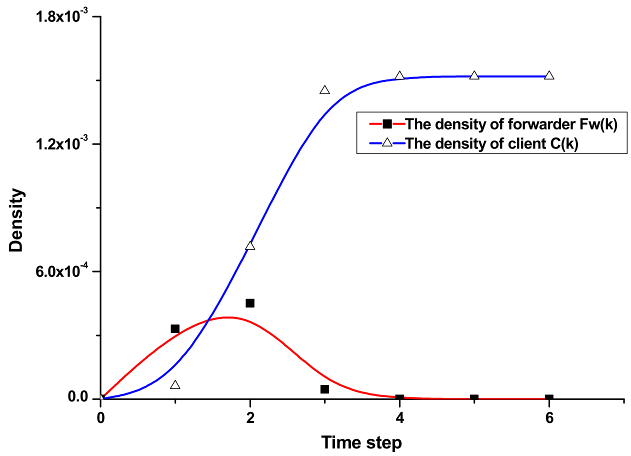

Fig. 12. Density variation of $Fw(t)$ and $C(t)$ with time step for the Tmall case.

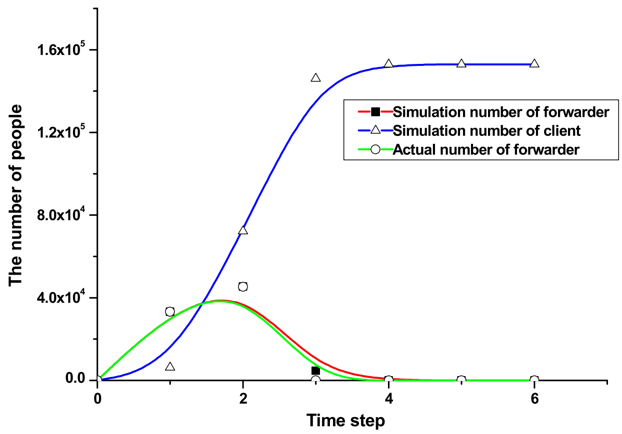

Fig. 13. Variation in number of forwarders and clients with time step for the Tmall case.

Table 4
Xiaomi phone forwarder data.

	Primary forwarders	Secondary forwarders	Tertiary forwarders
Number	49 421	2795	44

the smallest is 10. Fig. 14 shows the simulation results for the relationship between the density of forwarders and clients with forwarding time steps. Fig. 15 shows the simulation number of clients calculated by the FSFC model and the corresponding actual number of clients.

7.5. Discussion

The simulation results presented in Figs. 10, 12 and 14 have similar curves. In the early stages of preferential information spreading, there are numerous primary forwarders when the preferential information is published, which results in a rapid increase in the number of forwarders and a small number of secondary forwarders. Tertiary forwarding seldom occurs; hence, the number of forwarders who are converted to clients begins to decline following the third forwarding time step. Followers need to be infected in order to become clients, and there is a delay in the process. Consequently, the number of clients increases after the first forwarding time step. Further, the number of clients following the forwarders achieves steady state after the third forwarding time step.

The results shown in Figs. 11, 13 and 15 were obtained using the FSFC model, in which the spreading capacity of every node in the social network differs. The nodes comprise large-degree nodes and small-degree nodes, but preferential information spread from small-degree nodes to large-degree nodes is blocked. The simulation results in Table 5 show that the relative

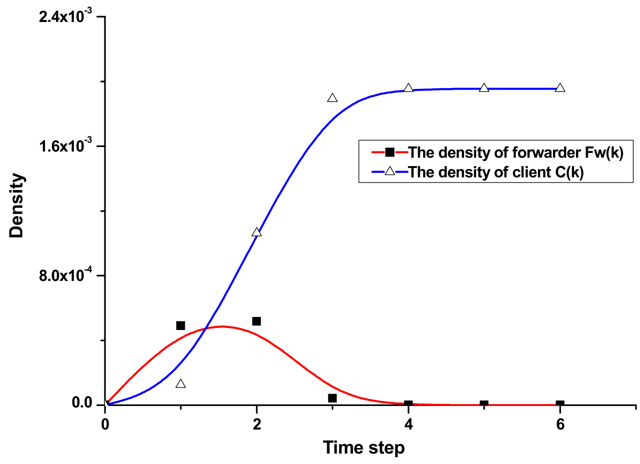

Fig. 14. Density variation of $Fw(t)$ and $C(t)$ with time step for the Xiaomi phone case.

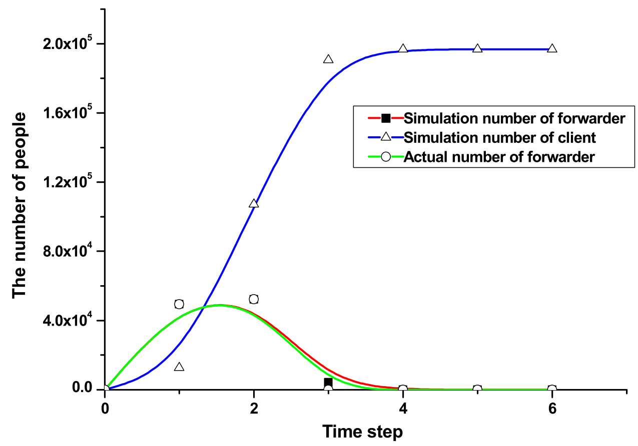

Fig. 15. Variation in number of forwarders and clients with time step for the Xiaomi phone case.

Table 5
Relative error of simulation results compared with actual results.

	Time step	Actual number	Simulation number for FSFC model	Relative error of FSFC model
Vivo smartphone	1 2	21 964 22 069	21 962 22 070	0.0068%
Tmall	1 2	33 226 45 342	33 224 45 347	0.0085%
Xiaomi phone	1 2	49 421 52 216	49 419 52 204	0.032%

error of the simulated results of the FSFC model compared with that of the actual data is low. This proves that the FSFC model is a feasible dynamic model for preferential information. During the simulations, we verified that our model can achieve good results in BA networks of various sizes, only the values of β_1 and β_2 need changing for it to adapt to different networks. However, we also observed that small-scale networks may cause greater errors owing to actual differences.

8. Conclusion

In this paper, we proposed a new dynamic model, called the follower super forwarder client (FSFC) model, to describe the dynamic behavior of preferential information spread via online social networks. The FSFC model, developed based on the SIR model, defines and distinguishes between large-degree and small-degree nodes. Further, the edge formed by a small-degree node and a large-degree node has a significant weight. In addition, two kinds of infection probabilities are adopted for large-degree node forwarders and small-degree node forwarders. Our simulation results verify that the FSFC model is a feasible dynamic model for preferential information. Because the current parameter values were not derived from the data, the model may have limitations in practical applications. Thus, in future work, we plan to optimize the key parameters of the FSFC model using more practical data.

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